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Stochastic modelling and control of comminution processes in jaw crushers

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ABSTRACT: For controlling of comminution processes in jaw crushers it is essential to build suitable model. One of possible concepts is the attempt of stochastic description, based on precisely planned experiments. In order to determine the type of dependence between grain composition of product and conditions of process course, it should be done the approximation of empirical distribution functions with using the formula:

$$1-\Phi(d)=F(d)=e^{-c\left(\frac{d}{d_{\max}-d}\right)^{n}}$$

where *d:* grain size, *c*, $d_{\max}$, *n:* parameters, $\Phi(d)$: grain size distribution function. It is possible to combine its parameters with material properties and technological characteristics of crushing machine. All of independent variables showed the significant influence on value of parameters. The obtained mathematical models may be used to choose the crusher work parameters in order to achieve the desirable compositions of crushing products of certain material. The results are possible to adapt to the industrial conditions, what was confirmed in conducted experiments.

1 INTRODUCTION

Comminution processes in different crushers are in practice very seldom described with using mathematical models, for the sake of both lack of methodical research over influence of both the change of machine's parameters and the type of crushed material on comminution effects (Brożek and others, 1995). Such models would enable us to work out principles of crushers' work controlling, namely a selection of parameters' value according to processed material in order to obtain desirable size analysis of product.

Receiving proper stochastic models combining comminution results with conditions of crusher's work and with profile of crushing material should be possible on the basis of experimental laboratory research. In order to forecast outcomes of industrial crushing on the basis of mentioned models it is essential to characterize somehow so-called a problem of the scale, that is principles of transferring laboratory crushing results into industrial scale. The article describes realization and effects of such organized research.

2 LABORATORY RESEARCH

The series of research started from jaw crusher, with simple move of jaw, upper-axial with off-centre power transmission. Smooth jaws were used, the dimensions of inlet: 110 by 145 millimeters, the width of outlet crack: 10÷35 millimeters, the angle of handle *a*: 20°. There were used five different materials in investigation, namely sandstone, limestone, dolomite, porphyry and diabase, specific of their geologic origin and physio-mechanical features.

For each material 27 breakings were planned: for three different widths of outlet crack *e*, for three different jumps *s* and for three different rotary speeds of drive shaft *f* (i.e. frequency of moving jaw's vibrations), for *f* = 300, 350 and 400 revolutions per minute. The scheme of th values of changeable parameters were presented in Table 1. Relationship between *e* and *s* were planned according to a dependence *(e+s)* from 17,7 to 38,4 millimeters (Gawenda 2004). According to literature (Kobiałka and others 2000), for small jaw crushers it is recommended using values of jump *s* equal from 0,3 to 0,5 of width of outlet crack. In order to increase the range of investigations in research presented in the article, the value of jump *s* was equaled from 0,18 to 0,53 of width of the outlet crack.

Table 1. A selection of outlet crack in crusher for a set of values of moving jaw's jump

Outlet crack *e* [mm]	Jump of moving jaw *s*, [mm]		
15	2,7	5,3	8,0
20	5,3	8,0	10,7
25	8,0	10,7	13,4

In order to characterize the type of dependence between size analysis of product and conditions of running of the process, approximations of empirical cumulative distribution functions with using the least squares method were carried out with the formula: (Cardu and others, 1993):

$$1-\Phi(d)=F(d)=e^{-c\left(\frac{d}{d_{\max}-d}\right)^{n}} \quad (1)$$

where: d – grain size, c, d_{max}, n – parameters, $\phi(d)$ - cumulative distribution function of grain size. In order to compute values of n and c parameters with using the least squares methods, at the beginning of computations it should be fixed the value of d_{max}. Next, such values of n and c parameters should be obtained, which minimize the rest deviation computed from the formula (2):

$$s_r=\sqrt{\frac{\sum_{i=1}^{p_s}\left(\Phi_e(d_i)-\Phi_t(d_i)\right)^2}{p_s-2}} \quad (2)$$

where: p - number of sieves used with size of mesh d_i; $\Phi_e(d_i)$ and $\Phi_t(d_i)$ - values of empirical cumulative distribution function and the one obtained from approximation formula for grain size d_i, respectively.

After initial calculations for all values of d_{max} parameter larger than the maximum mesh size (with the precision to 1 mm), to further calculations these values of n, c and d_{max} parameters were taken, for which the s_r value in formula (2) was mimimal.

If the approximation formula of size analysis curve of product is convergent with reality, it is then justified to tie its parameters with characteristic of material and with technological characteristic of crushing appliance. In case of jaw crusher it could be interesting following dependencies (for parameters of formula (1)):

$$k=f_1(d,w,P,e,s,f) \quad (3)$$

where:

k – specific parameter in formula (1) that is: n, c, d_{max} or d_{50} and d_{80}, and also yields of chosen size grade;

d – average graining of feed;

w – filling of working space;

P – durability characteristic of grains (for example Poisson's number; Protodiakonow index);

e – width of outlet crack in jaw crusher;

s – jaw's jump;

f – frequency of jaw's vibrations

3 RESULTS OF REGGRESIVE MODELLING OF CRUSHER'S WORK

Table 2 presents linear correlation coefficients among considered variables. Bold values in the table are significant in the significance level 0,05. From the point of view of mathematical modeling of comminution processes, in analysis of correlation coefficients the most significant correlations exist among independent variables (e, s, f, P) and dependent ones (n, c, d_{max}, d_{50}, d_{80}, γ_{-20}).

In classical factorial experiment, which gives best results in the range of linearity of correlation coefficients, it is assumed that coefficients of linear correlation between independent variables equal zero. In experiments run in the article above principle was kept with exception of linking the width of outlet crack and jaw's jump, for which correlation coefficient equals 0,71. It is caused by necessity of keeping technical conditions of crusher's work (s to e ratio shouldn't exceed 0,3÷0,5). Correlating of these values impinges on possibility of dividing their influences on crushing results, because some of linear regression coefficients at e or s in equations may become irrelevant for the sake of taking over of both impacts by coefficient at variable existing in the equation. Frequency of moving jaw's vibration f has practically no influence on values of coefficients in formula (1).

It is worth noticing that parameters of formula (1), namely: n, c and d_{max}, are strongly correlated with values of e and s. According to expectations values n and c are negatively correlated with e and s what means that with increasing in value of e and s a content of fine grains in the product is increasing. Value of d_{max} is positively correlated with e and s, what is obvious.

The similar situation presents correlation of values d_{50} and d_{80} with the yield of size grade under 20 mm γ_{-20} and technical (technological) parameters of crusher. For that reason a correlation of n, c and d_{max} parameters with values d_{50}, d_{80}, and γ_{-20} is evident. Protodiakonov consistence index P is in practice correlated only with the shape parameter n what denotes that with increasing in value of P the yield of fine size grades of product rises (harder materials give greater number of fine grains in comminution processes). According the to methodology of investigations over modeling of comminution processes in jaw crushers proposed in the article, there were calculated regressive dependencies combining parameters of crusher's work and Protodiakonov consistence index with parameters of formula (1). Next, also dependencies combining directly the yield of under 20 mm size grade in product, d_{50} and d_{80} grains with independent variables mentioned above were calculated.

Table 2. Matrix of linear correlation coefficients between considered variables (135 of data sets)

Variable	e	s	f	P	n	C	d_{max}	d_{50}	d_{80}	γ_{-20}
e, mm	1,00	**0,71**	0,00	0,00	**-0,58**	**-0,67**	**0,40**	**0,81**	**0,84**	**-0,88**
s, mm	**0,71**	1,00	0,00	0,00	**-0,52**	**-0,43**	**0,22**	**0,56**	**0,58**	**-0,59**
f, rev/min	0,00	0,00	1,00	0,00	0,04	0,09	0,04	-0,06	-0,05	0,03
P	0,00	0,00	0,00	1,00	**0,32**	0,00	0,11	0,05	0,00	-0,03
N	**-0,58**	**-0,52**	0,04	**0,32**	1,00	**0,78**	-0,02	**-0,65**	**-0,71**	**0,63**
C	**-0,67**	**-0,43**	0,09	0,00	**0,78**	1,00	0,10	**-0,83**	**-0,81**	**0,80**
d_{max}, mm	**0,40**	**0,22**	0,04	0,11	-0,02	0,10	1,00	**0,31**	**0,42**	**-0,35**
d_{50}, mm	**0,81**	**0,56**	-0,06	0,05	**-0,65**	**-0,83**	**0,31**	1,00	**0,98**	**-0,97**
d_{80}, mm	**0,84**	**0,58**	-0,05	0,00	**-0,71**	**-0,81**	**0,42**	**0,98**	1,00	**-0,97**
γ_{-20}, %	**-0,88**	**-0,59**	0,03	-0,03	**0,63**	**0,80**	**-0,35**	**-0,97**	**-0,97**	1,00

Modeling results (linear regressive models) obtained for all 135 experiments are presented below (numbers in square brackets denote errors of coefficients):

$$\hat{n} = \underset{[0,080]}{1,094} - \underset{[0,003]}{0,012}e - \underset{[0,003]}{0,009}s + \underset{[0,03]}{0,014}P; \quad R=0,68287$$

$$\hat{c} = \underset{[0,473]}{3,336} - \underset{[0,015]}{0,126}e + \underset{[0,020]}{0,018}s + \underset{[0,001]}{0,001}f; \quad R=0,68016$$

$$\hat{d}_{max} = \underset{[5,299]}{29,359} + \underset{[0,171]}{0,749}e - \underset{[0,224]}{0,251}s + \underset{[0,012]}{0,006}f + \underset{[0,186]}{0,261}P;$$
$R=0,42902$

$$\hat{d}_{50} = -\underset{[2,437]}{0,688} + \underset{[0,079]}{0,912}e - \underset{[0,103]}{0,040}s - \underset{[0,006]}{0,006}f + \underset{[0,085]}{0,090}P;$$
$R=0,81639$

$$\hat{d}_{80} = \underset{[3,030]}{3,273} + \underset{[0,098]}{1,258}e - \underset{[0,1288]}{0,062}s - \underset{[0,007]}{0,007}f + \underset{[0,106]}{0,010}P;$$
$R=0,84181$

$$\hat{\gamma}_{-20} = \underset{[7,943]}{135,787} - \underset{[0,246]}{3,811}e + \underset{[0,323]}{0,349}s + \underset{[0,017]}{0,0111}f - \underset{[0,268]}{0,194}P;$$
$R=0,8796$

Best modeling results for parameter n were obtained for limestone and dolomite – the highest value of correlation coefficient R, the lowest values of rest deviation coefficient s_r.

It is also worth noticing that all independent variables reveal significant influence on value of n parameter. In all equations obtained for individual materials a jaw's jump and frequency of it's vibrations have no relevant influence on the value of n and the only decisive parameter is the width of outlet crack of the crusher.

In the model describing behaviour of all materials the frequency of moving jump's vibration turned out to be irrelevant, what may be connected with the character of its motion (a strait movement of jaw). It is necessary to emphasize that in crushers with compound movement of jaw, the frequency has greater influence on graining of comminution products for the sake of presence of abrasion effect (drawing the material into the depth of chamber). The quality of model is very high, and a significant influence of Protodiakonov consistence index P is worth emphasizing (Protodiakonov and others, 1957).

Analogical regressive models for c and d_{max} parameters from formula (1) have totally different characteristics. The only value having significant influence on these parameters is the width of outlet crack of crusher, while the dependence is negative for c and positive for d_{max}. It denotes that with increasing in e value the value of c is decreasing, while the value of d_{max} is increasing. Parameter c is called a scale parameter. Similar characteristic has d_{max} parameter, because both ones describing a range of changeability of grain sizes in comminution product. It is then evident that frequency of moving jaw's vibration – increase of which should cause the rise of fine grains – has no greater influence on parameters c and d_{max}. The influence of frequency is observable only for limestone and, in some sense, for diabase.

In regression equations describing comminution of all materials there always exists significantly width of outlet crack. The influence of remaining independent variables is differentiated. For the reason that moving jaw's jump is in some sense selected to the outlet crack, its influence is less noticeable (coefficients at *e* value take over part of influence of *s*). Significant influence of *s* value exists only in regression equations for *n* parameter. In these and only equations a significant influence of Protodiakonov consistence index, the only index characterizing physio-chemical properties of material, is also noticeable. In remaining equations, especially in those modeling directly comminution results for d_{50}, d_{80} and γ_{-20}, the influence of Protodiakonov consistence index is negligible. It should be admitted that multiple correlation coefficients for all equations are essential on the significance level $\alpha<0,05$ what confirms legitimacy of linear regressive models.

In order to characterize additionally the influence of materials' quality on comminution effects there were computed regression equations with using apparent variables. It consist in introducing of four additional columns of independent variables *Q1, Q2, Q3* and *Q4* into a data matrix. These columns would characterize rock materials with the aid of sequence of zero and one numbers. All experiments for limestone have system 0, 0, 0, 0; for sandstone 1, 0, 0, 0; for dolomite 0, 1, 0, 0; for porphyry 0, 0, 1, 0 and for diabase 0, 0, 0, 1. In this manner there were obtained following equations:

$$\hat{n} = \underset{[0,063]}{1,259} - \underset{[0,002]}{0,012}e - \underset{[0,003]}{0,008}s - \underset{[0,019]}{0,104}Q_1 - \underset{[0,019]}{0,072}Q_3 + \underset{[0,019]}{0,058}Q_4$$

R=0,7979

$$\hat{c} = \underset{[0,434]}{3,569} - \underset{[0,015]}{0,122}e - \underset{[0,134]}{0,427}Q_1 - \underset{[0,134]}{0,339}Q_3\,; \quad R=0,7134$$

$$\hat{d}_{\max} = \underset{[5,03]}{31,45} + \underset{[0,171]}{0,749}e\,; \quad R=0,4501$$

$$\hat{d}_{80} = \underset{[2,69]}{1,14} - \underset{[0,091]}{1,258}e - \underset{[0,83]}{3,73}Q_1 + \underset{[0,83]}{2,59}Q_2 + \underset{[0,83]}{2,48}Q_3 + \underset{[0,83]}{2,37}Q_4$$

R=0,8663

$$\hat{d}_{50} = -\underset{[2,17]}{1,58} + \underset{[0,07]}{0,91}e - \underset{[0,67]}{2,80}Q_1 + \underset{[0,67]}{2,27}Q_2 + \underset{[0,67]}{1,57}Q_3 + \underset{[0,67]}{2,52}Q_4$$

R=0,8450

$$\hat{\gamma}_{-20} = \underset{[6,95]}{138,60} - \underset{[0,23]}{3,81}e - \underset{[2,15]}{7,69}Q_1 - \underset{[2,15]}{5,67}Q_2 - \underset{[2,15]}{4,46}Q_3 - \underset{[2,15]}{6,41}Q_4$$

R=0,8908.

In square brackets under regression coefficients there are presented statistical errors. All regression equations are significant (values of *R* are presented under equations). The interpretation of equations with apparent variables is as follow: the part of equation connected with parameters of crusher's work characterizes general principle of crushing process, while the part connected with apparent variables allows to obtain of five equations, in which only the absolute term is changing. The change results from adding to the absolute term a suitable coefficient standing at apparent variable. For equation describing for example d_{50} value we obtain:

$d_{50} = -1,58 + 0,91e$ – for limestone
$d_{50} = -1,58 + 0,91e - 2,80$ – for sandstone
$d_{50} = -1,58 + 0,91e + 2,27$ – for dolomite
$d_{50} = -1,58 + 0,91e + 1,57$ – for porphyry
$d_{50} = -1,58 + 0,91e + 2,52$ – for diabase

Significant coefficients at apparent variables appeared in four regression equations (an error of coefficient is at least two times lower than its value). In case of *n* parameter coefficients have different signs, what proves the possibility of change the swelling of cumulative distribution function in dependence on material. In case of *c* parameter only the sandstone and porphyry noticeably change (decrease) its values. For d_{50} and d_{80} parameters it occurs major or minor increase in their values in comparison to comminution of limestone. We can also claim that approximation results of d_{max} depend only on the width of outlet crack of the crusher and the type of material plays no role here. In modelling of γ_{-20} yields it turned out that all materials increase in yield of –20 mm size grade in comparison to limestone. Significance level of equation is increasing and coefficients in base part of equation (absolute term and coefficient at *e* parameter) haven't changed in comparison to prior analogous equation.

4 CONCLUSIONS

Obtained modelling results, with using both Protodiakonov consistence index and apparent variables, lead up to conclusion that attempt at representing of material's quality with using a single number in statistical modelling comes to failure. It is caused by the fact that indexes of material quality in comminuted set of grains have also statistical character and should be represented by their own distribution (Hudson, 1969). Grain analysis of crushing product, according to Rittinger's theory, is connected with comminution energy. The comminution energy during disintegration of the grain is dependent on stretching durability, and the one is next dependent on the structure of grain.

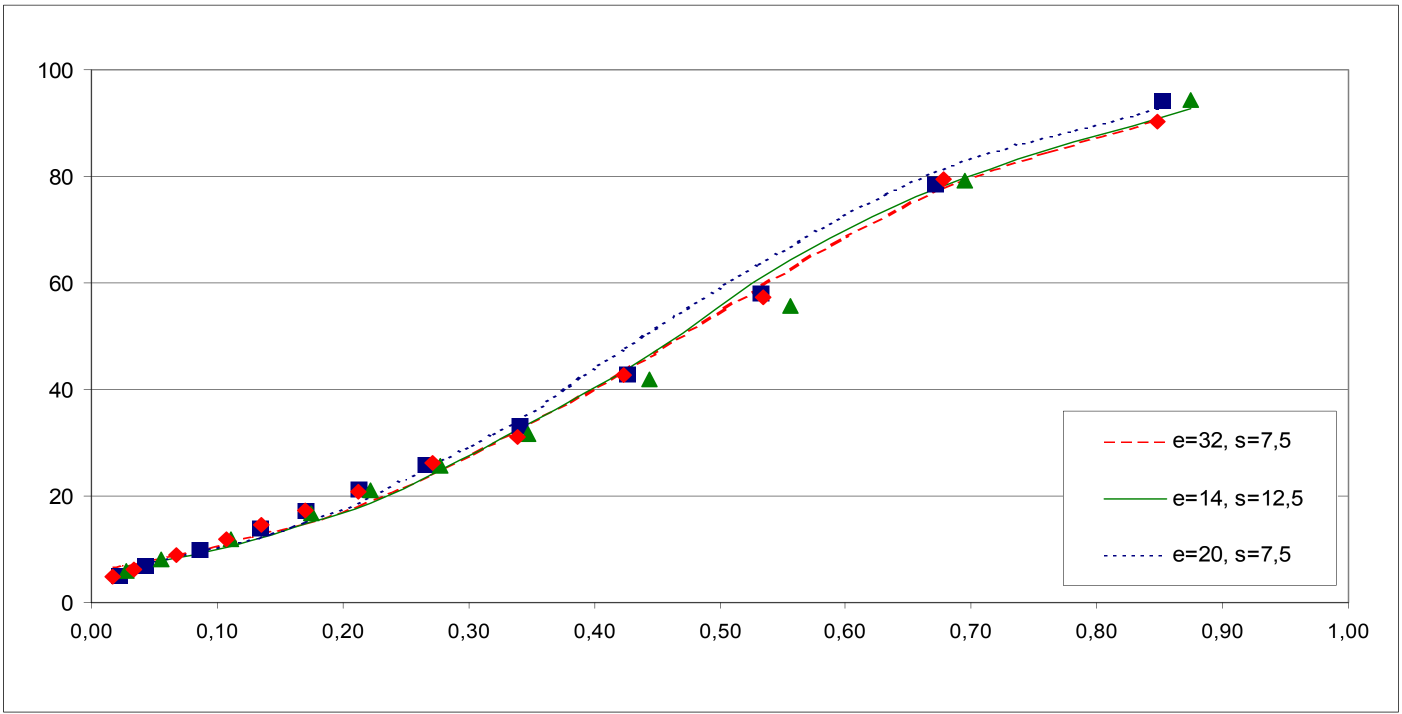

Figure 1. Comminution results of chalcedonite in jaw crusher

Following qualitative parameters of material then affect to comminution process: texture or structure, distribution of cracks etc. (Zawada, 1998). For that reason a statistical modelling of comminution parameters may succeed if noticeable results would be achieved either in stereological research (Unland, 2000; Unland and others, 1998) or in those connected with group or laminar comminution (Szczepinski, 1974; Dresher, 1983).

In order to take advantage of obtained mathematical models in achieving of demanded graining (size analysis) of comminuted products by a selection of proper crusher's work parameters, it should be assumed a prior given cumulative distribution function (in the sense of formula (1) parameters). After determining of Protodiakonov consistence index, it should be fixed the width of outlet crack resulting from assumed d_{max} and then the moving jaw's jump should be such selected in order to obtain assumed swelling of cumulative distribution function characterised by n value. Experiments presented in the article were laboratory ones and their transfer into industrial scale can be realised with using current assumptions frequently existing in matrix models. It is assumed there that regardless of the size of comminuted grains a comminution matrix is obtained by subtracting value

$$\left(\frac{d_i}{D_{\max}}\right)^k$$

for two consecutive sieves, and D_{max} is maximum grain size of feed. For that reason a quotient

$$\frac{d_i}{D_{\max}}$$

accepts values from range (0,1).

It was confirmed thanks using in investigations the smallest industrial semi-mobile jaw crusher in Poland, type L44.41 produced by Makrum in Bydgoszcz. The crusher has an easy adjustment of following parameters: jaw's jump (7,5÷12,5 mm), width of the outlet crack (8÷32 mm), angle of inclination of strut plate (to 20 ^{o}), rotational speed of shaft (60÷400 revolutions per minute) and has possibility of assembling of crushing jaws with different shapes of toothing.

There were crushing different size grades of chalcedonite with assuming the principle that e parameter equals 1/3 of average grain size of feed. Comminution results were approximated by formula (1) discussed above. Thanks to determining d_{max} value by approximation method it was possible to present obtained curves in (d/d_{max}, $\Phi(d)$) co-ordinates system (fig.1).

It appeared that these curves superimpose what means that it can be accepted following procedure in transferring of laboratory experiments into industrial scale:

- to determine roughly d_{max} value in formula (1) based on regressive dependency linking d_{max} with e parameter,
- to calculate the yield of product size grades for industrial crushing on the basis of laboratory obtained curve presented in (d/d_{max}, $\Phi(d)$) co-ordinates system.

It is worth noticing the fact that applying the formula (1) for coarse comminution is undisputed.

Furthermore, its applying for transferring of laboratory results into industrial scale is the subject of further investigations led by authors.

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